22533

V Semester B.Sc. Examination, November/December 2014
(Semester Scheme)
MATHEMATICS (Paper – V)

Time : 3 Hours Max. Marks : 90

Instructions : *1) Answer **all** questions.*
*2) Answer should be written **completely** in **English**.*

I. Answer **any fifteen** of the following : **(15×2=30)**

1) Define :
   i) Ring with unity
   ii) Division ring

2) In a ring (R, +, .) prove that $(-a)(-b) = ab \ \forall\ a, b \in R$.

3) Show that $M = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \Big/ a, b \in z \right\}$ of all 2×2 matrices is a right ideal of ring R of 2×2 matrices over z.

4) With an example show that a subring of a non-commutative ring is commutative.

5) State fundamental theorem of homomorphism of rings.

6) If $f : R \to R'$ is a homomorphism prove that $f(-a) = -f(a)$.

7) If $\vec{r} = \cos \omega t\ \hat{i} + \sin \omega t\ \hat{j}$ where ω is a constant, prove that $\vec{v}$ is perpendicular to $\vec{r}$.

8) Prove that $\frac{d\vec{A}}{dt} \cdot \vec{A} = 0$ of $|\vec{A}|$ is constant.

9) Find the unit tangent vector at any point for the curve $x = \frac{1}{2} \sin s$, $y = \frac{1}{2} \cos s$, $z = \frac{\sqrt{3}}{2} s$ where s is the arc length.

P.T.O.

10) Show that the necessary condition for a space curve to be a plane curve is $\left[\vec{r^{I}}\ \vec{r^{II}}\ \vec{r^{III}}\right]=0$.

11) Find the spherical co-ordinates of the point whose Cartesian co-ordinates are $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}, -1\right)$.

12) Find the maximum directional derivative of $\phi = xy^2 + yz^2 + zx^2$ at (1, 1, 1).

13) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ find curl $\vec{r}$.

14) Prove that div (curl $\vec{F}$) = 0.

15) Show that $\nabla^2\left(\frac{1}{r}\right) = 0$, where $r = |\vec{r}|$.

16) If $\vec{F}$ and $\vec{G}$ are irrotational, then prove that $\vec{F} \times \vec{G}$ is solenoidal.

17) Prove that $P_n(-x) = (-1)^n P_n(x)$.

18) State the orthogonal property of Legendre polynomial.

19) Write the Bessel's differential equation.

20) Show that $\frac{d}{dx}[x J_1(x)] = x J_0(x)$.

II. Answer **any four** of the following : **(4×5=20)**

1) Prove that the ring $(Z_n, \pm_n, X_n)$ is an integral domain iff n is a prime number.

2) If R is a ring such that $a^2 = a, \forall a \in R$ then show that :

i) $a + a = 0$

ii) $a + b = 0 \Rightarrow a = b$

iii) R is a commutative ring

3) Prove that a non-empty subset s of a ring R is a subring of R iff :

i) $\forall\, a, b \in s \Rightarrow a - b \in s$

ii) $\forall\, a, b \in s \Rightarrow ab \in s$

4) Prove that the intersection of any two ideals of a ring is again an ideal of the ring.

5) Define Kernel of a ring homomorphism, if $f : R \to R'$ is a homomorphism, then show that Ker f is an ideal of R.

6) If $f : R \to R'$ defined by $f(a + ib) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ where (R, +, .) is a ring of all complex numbers and R' is a ring of all 2×2 matrices, prove that f is an isomorphism.

III. Answer **any three** of the following : **(3×5=15)**

1) Prove the Serret-Frenet formulae in the form $t' = \vec{d} \times \hat{t}$, $n' = \vec{d} \times \hat{n}$ and $b' = \vec{d} \times \hat{b}$ where $\vec{d} = \tau\,\hat{t} + k\,\hat{b}$.

2) For the space curve x = a cost, y = a sint, z = bt, find K and τ.

3) For the space curve $x = \tan^{-1} s$, $y = \frac{1}{\sqrt{2}} \log (s^2 + 1)$ and $z = s - \tan^{-1} s$, find $\hat{t}$ and $\hat{n}$.

4) Find the normal vector and tangent plane to the cylinder $x^2 + y^2 = 4$ at the point $(1, \sqrt{3}, 2)$.

5) Express $\vec{f} = 3y\,\hat{i} + x^2\,\hat{j} - z^2\,\hat{k}$ in cylindrical co-ordinates.

IV. Answer **any three** of the following : **(3×5=15)**

1) Show that $\nabla [\vec{r}\ \vec{a}\ \vec{b}] = \vec{a} \times \vec{b}$, where $\vec{a}, \vec{b}$ are constant vectors.

2) Show that $\operatorname{div}\left\{\frac{f(r)}{r}\vec{r}\right\} = \frac{1}{r^2}\frac{d}{dr}\left\{r^2 f(r)\right\}$.

3) Find the constants a, b, c, so that $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational.

4) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find $\nabla^2 r^{n+1}$.

5) If $\phi(x, y, z)$ be a scalar-point function and $\vec{F}$ is a vector-point function, then prove that $\operatorname{div}(\phi\vec{F}) = \phi(\operatorname{div}\vec{F}) + (\operatorname{grad}\phi).\vec{F}$.

V. Answer **any two** of the following : **(2×5=10)**

1) Express $x^3 - 3x^2 + x$ in Legendre's polynomial.

2) Show that $x P_n'(x) - P_{n-1}'(x) = n P_n(x)$.

3) Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

4) Show that $\cos(x\cos\theta) = J_0 - 2\cos 2\theta\ J_2 + 2\cos 4\theta\ J_4 - + \ldots\ldots$

Q.P. Code – 42533

Fifth Semester B.Sc. Degree Examination, October/November 2018

(CBCS – Semester Scheme)

Mathematics

Paper 5.1 — ADVANCED ALGEBRA AND NUMERICAL METHODS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1. *Answer ALL questions.*
2. *Answers should be written completely in English.*
3. *Scientific calculators are allowed.*

PART – A

I. Answer any **SIX** questions : **(6 × 2 = 12)**

1. Prove that a ring is without zero divisors if the cancellation law holds in it.

2. If $f : R \to R'$ is homomorphism of two rings, then prove that $f(0) = 0'$ where 0 and $0'$ are zeros of R and R' respectively.

3. Is z an ideal of Q? Where $(z + 0)$ is the ring of integers and $(Q, + 0)$ the ring of rational number. Substantiate your answer.

4. Show that the set $\{(1, 2, 1)(3, 4, -7)(3, 1, 5)\}$ is Linearly independent.

5. Define linear transformation of vectorspaces.

6. Explain Regula-Falsi method to solve $f(x) = 0$.

7. Solve $\dfrac{dy}{dx} = x + y$ by Euler's Method given that $y = 1$ when $x = 0$, $h = 0.2$.

PART – B

II. Answer any **SIX** questions : **(6 × 3 = 18)**

1. Give an example of a finite integral domain.

2. In a ring R such that $a + a = 0 \ \forall \ a \in R$, prove that R is commutative.

3. Prove that kernel of homomorphism of ring is an ideal.

4. $W = \{ (x_1, x_2, x_3) : x_1^2 + x_2^2 + x_3^2 \leq 0 \}$ a subset of the vector space $V_3(R)$. Check whether W a subspace of $V_3(R)$ or not.

5. Show that the set $S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ form a basis of the vector space $V(R)$ of all 2×2 matrices over R.

6. Find the root of the equation $2x - \log_{10}^x = 7$ which lies between 3.5 and 4 by Regula-Falsi method.

7. Solve by Jacobi iteration method $5x - y + 3z = 10$; $36x + 6y = 18$; $x + y + 5z = -10$, taking $(3, 0, -2)$ as initial approximation to the solution upto two iterations.

PART – C

III. Answer any **FOUR** questions : **(4 × 5 = 20)**

1. Prove that every field is an Integral Domain. Is the converse true? Give an example.

2. Prove that a non empty subset S of a ring R is a subring if and only if

   (a) $S + (-S) = S$

   (b) $SS \subseteq S$

   where $-S$ is the set of all negative elements of S.

3. Show by an example that a ring without unity, may have a subring with unity.

4. Prove that the Quotient ring R/I, is the homomorphic image of the ring R with I as its Kernel.

5. $(z + 0)$ be the ring of integers and $(2z + *)$ where $*$ on $2z$ defined by $a * b = \frac{ab}{2}$ $\forall\, a, b \in z$, the ring of even integers. Check whether the map defined $f : z \to 2z$ by $f(x) = 2x \;\forall\; x \in z$ is an isomorphism.

IV. Answer any **FOUR** questions : **(4 × 5 = 20)**

6. Define a vectorspace over a scalar field

Show that $V = \left\{ \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} : x, y \in R \right\}$ is a vectorspace over the real scalar field R.

7. Prove that intersection of two subspaces of a vectorspace is a subspace. Is the statement true in the case of union of two subspaces? When the union of two subspaces is a subspace?

8. Check whether the vector $\alpha = \begin{pmatrix} 3 & -1 \\ 1 & -2 \end{pmatrix}$ of the vectorspace $M_2(R)$ of all 2×2 matrices over the field of reals R is in the linear combination of $\alpha_1 = \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$ and $\alpha_3 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$.

9. For the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, find the corresponding linear transformation $T : R^2 \to R^2$ with respect to the basis $\{(1, 0)(1, 1)\}$.

10. State and prove Rank-Nullity theorem.

V. Answer any **FOUR** questions : **(4 × 5 = 20)**

11. Find a real root of the equation $x^3 - x - 1 = 0$ using Bisection method in four stages.

12. Solve the following equations

$20x + y - 2z = 17$; $3x + 20y - z = -18$ and $2x - 3y + 20z = 25$ by Jacobi method.

13. Find the cube root of 25 correct to 3 places of decimal by Newton-Raphson method.

14. For the equation $\frac{dy}{dx} = f(x, y)$ write upto third approximation by Picard's method.

15. Find the approximate solution $x = 1.2$ of the equation $\frac{dy}{dx} = xy$ given $y(1) = 2$ by Runge-Kutta method.

14/11/16 A

22533

V Semester B.Sc. Examination, Nov./Dec. 2016
(Semester Scheme)
MATHEMATICS (Paper – V)

Time : 3 Hours Max. Marks : 90

***Instructions** : 1) Answer **all** the questions.*
*2) Answer should be written **completely** in **English**.*

I. Answer **any fifteen** of the following : **(15×2=30)**

1) Define a commutative ring and give an example.

2) Find all zero divisors of the ring $(Z_8, +_8, \times_8)$.

3) Prove that the set $5z = \{5n / n \in z\}$ is an ideal of a ring $(z, +, \bullet)$.

4) Define homomorphism and isomorphism of rings.

5) Show that a mapping $f : (z, +, \bullet) \to (2z, +, \bullet)$ defined by $f(n) = 2n \; \forall \; n \in (z, +, \bullet)$ is not homomorphism, where $(z, +, \bullet)$ and $(2z, +, \bullet)$ are rings.

6) Define Quotient ring.

7) If $\vec{r} = \vec{a}\cos wt + \vec{b}\sin wt$, 'w' is constant. Show that $\vec{r} \times \frac{d\vec{r}}{dt} = w(\vec{a} \times \vec{b})$.

8) If $\vec{a} = t^2\hat{i} + \text{Log}t\,\hat{j} + (3t+7)\hat{k}$ and $\vec{b} = 3t\hat{i} + 7e^t\hat{j} + 2t\hat{k}$. Find $\frac{d}{dt}(\vec{a}.\vec{b})$.

9) Define radius of curvature and radius of torsion of a curve.

10) For the space curve $x = 3\cos t$, $y = 3\sin t$, $z = 4t$. Find direction cosine's of tangent.

11) Find the cylindrical coordinates of a point whose Cartesian coordinates are (2, 2, 4).

12) Find the unit normal vector to the surface $2x^2 + 4yz = 5z^2 - 10$ at (3, –1, 2).

13) Evaluate grad (e^{r^n}), where $r^2 = x^2 + y^2 + z^2$.

14) If $\vec{a} = \frac{y\hat{j} - x\hat{i}}{x^2 + y^2}$, find div$(\vec{a})$.

15) Find $\nabla^2\phi$, if $\phi = x^2 - y^2 + z$.

16) If $\vec{F} = (6xy + z^3)\,\hat{i} + (3x^2 - z)\,\hat{j} + (3xz^2 - y)\,\hat{k}$. Find Curl$\vec{F}$.

P.T.O.

17) Prove that $1 + x + x^2 = \frac{1}{3}[2p_0(x) + 3p_1(x) - 2p_2(x)]$.

18) Show that $P_n'(1) = \frac{n(n+1)}{2}$.

19) Show that $J_{\frac{1}{2}}\left(\pi/2\right) = \frac{2}{\pi}$.

20) Using the recurrence relation for $J_n(x)$, show that $\frac{d}{dx}\left(\frac{J_1(x)}{x}\right) = -\frac{J_2(x)}{x}$.

II. Answer **any four** of the following : **(4×5=20)**

1) Prove that every field is an integral domain. Is the converse true ?

2) Show that the set of all matrices of the form $\begin{pmatrix} a & b \\ o & c \end{pmatrix}$ is a subring of the ring of 2×2 matrices with integral elements, with respect to matrix addition and multiplication.

3) Prove that field has no proper ideals.

4) If $f : R \to R'$ is a homomorphism of ring R onto ring R′ with Kernel 'K' then prove that 'Kernel K' is a subring of R and 'Kernel K' is an ideal in R.

5) Show that the mapping $f : R \to R$ defined by $f(a + b\sqrt{3}) = a - b\sqrt{3}$ is a homomorphism. Where $R = \{a + b\sqrt{3} / a, b \in z\}$ is a ring with respect to addition and multiplication. Find 'Kernel f'.

6) If $f : R \to R'$ is an isomorphism and R is an integral domain, then show that 'R′' is also an integral domain.

III. Answer **any three** of the following : **(3×5=15)**

1) Derive Serret-Frenet formulae for the curve $\vec{r} = \vec{r}(s)$.

2) Find $\hat{t}, \hat{n}$ and $\hat{b}$ for the space curve $x = t, y = t^2, z = t^3$ at $t = 1$.

3) Find the curvature and torsion of the curve $x = e^t, y = e^{-t}, z = t\sqrt{2}$ at any point on the curve.

4) Find the equation of the tangent plane and normal line to the surface $x^2 + y^2 = z + 3$ at $(2, -1, 2)$.

5) Prove that the spherical coordinate system is an orthogonal curvilinear coordinate system.

IV. Answer **any three** of the following : **(3×5=15)**

1) Find the directional derivative of $\phi(x, y, z) = x \sin z - y \cos z$ in the direction of the vector $2\hat{i} - 2\hat{j} + \hat{k}$ at the origin. Also find maximum directional derivative.

2) Show that $\text{div}(\text{grad } r^m) = m(m+1) r^{m-2}$. Where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

3) Show that vector $\vec{F} = (\sin y + z\cos x)\hat{i} + (x\cos y + \sin z)\hat{j} + (y\cos z + \sin x)\hat{k}$ is irrotational. Find 'ϕ' such that $\vec{F} = \nabla\phi$.

4) Show that the vector $\frac{\vec{r}}{r^3}$ is both solenoidal and irrotational, where '$\vec{r}$' is the position vector of p(x, y, z).

5) If 'ϕ' is a scalar function and '$\vec{A}$' is a vector function, then prove that $\text{curl}(\phi\vec{A}) = \phi(\text{curl}\vec{A}) + (\text{grad}\phi) \times \vec{A}$.

V. Answer **any two** of the following : **(2×5=10)**

1) Show that $\frac{1+t}{\sqrt{1-2xt+t^2}} = 1 + \sum_{n=0}^{\infty}[P_n(x) + P_{n+1}(x)]t^{n+1}$.

2) Show that $\int_{-1}^{1} xP_n(x)P_{n-1}(x)dx = \frac{2n}{4n^2-1}$.

3) Prove the relation $2nJ_n(x) = x[J_{n-1}(x) + J_{n+1}(x)]$.

4) Show that $J_{\frac{-3}{2}}(x) = -\sqrt{\frac{2}{\pi x}}\left[\frac{\cos x}{x} + \sin x\right]$.

Q.P. Code – 22533

Fifth Semester B.Sc. Degree Examination, November 2017

(Semester Scheme)

Mathematics

Paper V – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1) *Answer ALL questions.*

2) *Answers should be written completely in English.*

I. Answer any **FIFTEEN** of the following : **(15 × 2 = 30)**

1. Define a commutative ring without unity. Give an example.

2. Define an integral domain. Give an example.

3. If R is a ring without zero divisors, then prove that cancellation law holds in it.

4. Show that $S = \left\{ \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} \middle| \, a, b, c \in Z \right\}$ is a subring of the ring $M_2(Z)$ of all 2×2 matrices over Z.

5. If I is an ideal of a ring R with unity and $1 \in I$, then prove that $I = R$.

6. Show that the mapping $f : (Z, +, 0) \to (2Z, +, 0)$ defined by $f(x) = 2x \;\; \forall \; x \in Z$ is not an homomorphism.

7. If $\vec{r} = \vec{a} \cos wt + \vec{b} \sin wt$, show that $\vec{r} \times \frac{d\vec{r}}{dt} = w(\vec{a} \times \vec{b})$.

8. Show that $[r' \, r'' \, r'''] = k^2 \tau$ for the curve $\vec{r} = \vec{r}(s)$ where dashes derivative with respect to the arc length s.

9. If $\vec{f}(t)$ is a vector function of constant magnitude, then prove that $\vec{f} \cdot \frac{d\vec{f}}{dt} = 0$.

10. The cylindrical polar co-ordinates of a point are (3, 60°, 5). Find its Cartesian co-ordinates.

11. If a curve is a straight line then prove that $K = 0$ at all points.

12. Find $\nabla\phi$ at (2, – 1, 1) where $\phi = xy^2 + yz^3$.

13. Find the directional derivative of $\phi(x, y, z) = x\sin z - y\cos z$ in the direction of $2\hat{i} - 2\hat{j} + \hat{k}$ at the origin.

14. Show that $grad(\vec{r} \cdot \vec{a}) = \vec{a}$.

15. Find $div(grad\,\phi)$ if $\phi = x^2y^3z$.

16. Find $grad\,r$, where $r = |\vec{r}|$.

17. Prove that $P_n(-x) = (-1)^n P_n(x)$.

18. Define Legendre's and Bessel equations.

19. Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x$.

20. Show that $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$.

II. Answer any **FOUR** of the following : **(4 × 5 = 20)**

1. If R is a ring such that $a^2 = a \;\; \forall\, a \in R$, then show that

   (a) $a + a = 0$

   (b) $a + b = 0 \Rightarrow a = b$

   (c) R is a commutative ring.

2. Prove that intersection of any two subrings of a ring R is a subring of R.

3. (a) Define the ideal of a ring R.

   (b) Show that every ideal of a ring R is a subring of R.

   (c) Is every subring of R an ideal of R? Justify your answer.

4. If $f : R \to R'$ be a homomorphism of the ring R into the ring R', then prove that $f(R)$ is a subring of R'.

5. If I is an ideal of the ring R, prove that the quotient ring R/I is a homomorphic image of R with I as its kernel.

6. Prove that every finite integral is a field.

III. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. Derive Serrette-Frenette formula for the curve $\vec{r} = \vec{r}(u)$.

2. Find the angle between the tangents to the curve $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$ at $t = \pm 1$.

3. Show that Serrette-Frenette formula can be written in the form $\frac{d\hat{t}}{ds} = \vec{d} \times \hat{t}$, $\frac{d\hat{n}}{ds} = \vec{d} \times \hat{n}$, $\frac{d\hat{b}}{ds} = \vec{d} \times \hat{b}$ and determine the vector $\vec{d}$.

4. Find the unit normal vector, the equation of the tangent plane, and the normal line at $(1, -1, 2)$ for the surface $z = x^2 + y^2$.

5. Prove that cylindrical polar co-ordinate system is an orthogonal curvilinear co-ordinate system.

IV. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. Find the angle between the surfaces $3x^2 - 2y^2 - 3z^2 + 8 = 0$ and $4x^2 + 9y^2 = 40z$ at $(-1, 2, 1)$.

2. Show that $div\left[\frac{f(r)}{r}\vec{r}\right] = \frac{1}{r^2}\frac{d}{dr}(r^2 f(r))$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$.

3. For continuously differentiable Scalar function ϕ and Vector function $\vec{f}$, show that $\text{curl}(\phi\vec{f}) = \phi\,\text{curl}\,\vec{f} + \text{grad}\,\phi \times \vec{f}$.

4. Show that $r^n\vec{r}$ is irrotational for all values of n and solenoidal for $n = -3$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $\vec{r} = |\vec{r}|$.

5. Express $\vec{f} = 3y\hat{i} + x^2\hat{j} - z^2\hat{k}$ interms of cylindrical polar co-ordinates.

V. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. Prove that $\int_{-1}^{1} P_m(x) \cdot P_n(x)\, dx = 0$ if $m \neq n$.

2. Show that $\dfrac{1+z}{z\sqrt{1-2xz+z^2}} - \dfrac{1}{z} = \sum_{n=0}^{\infty} (P_n + P_{n+1}) z^n$.

3. Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.

4. Prove that $J_n(x) = \dfrac{x}{2n}[J_{n-1}(x) + J_{n+1}(x)]$.

Q.P. Code – 22534

Fifth Semester B.Sc. Degree Examination, November 2017

(Semester Scheme)

Mathematics

Paper VI – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1) *Answer ALL the questions.*
2) *Answers should be written completely in English.*

I. Answer any **FIFTEEN** of the following : **(15 × 2 = 30)**

1. Find the partial differential equation from $Z = (x+a)(y+b)$.

2. Solve, $z^2yp + z^2xq = xyz$.

3. Solve, $\sqrt{p} + \sqrt{q} = 1$.

4. Solve, $p^2 + q^2 = z$.

5. Solve, $[D^3 - 3D^2D' + 2D(D')^2]Z = 0$.

6. Prove that $(1+\Delta)(1-\nabla) = 1$.

7. Evaluate $\Delta(\log 3x)$.

8. Find the 7th term of the series 7, 15, 35, 72, 131, 217, ...

9. State Newton's divided difference formula for unequal intervals.

10. Evaluate $\int_0^6 y\,dx$ using Trapezoidal rule from the following table.

x	0	1	2	3	4	5	6
y	0.146	0.161	0.176	0.190	0.204	0.217	0.230

11. Find the K.E. of the particle of mass '8' moving with velocity $3i - 2j + 5k$.

12. A particle is moving with SHM. At what distance from the centre will the velocity be half that of maximum velocity?

13. The position of a moving particle in a straight line is given by $x = a\cos nt + b\sin nt$. Show that the motion is simple harmonic.

14. Derive the relation $5T^2 = 4H$, where H is the greatest height in mts reached by a projectile and T is the time of flight.

15. A hill has inclination of 30°. A ball is thrown with a given velocity, so as to have the maximum range up the hill. Prove that the angle at which it strikes the hill is 60°.

16. A particle describes a circle of radius r with a uniform speed 'V'. Show that its acceleration at any point of the path is $\frac{V^2}{r}$.

17. A particle moves so that its radial velocity at a point of its path is $2\lambda a\theta$ and its transverse velocity is λ. Find the radial acceleration.

18. A particle describes a circle with respect to the centre of the circle. Find the transverse acceleration of the particle.

19. Find the law of force for the curve $P = r\sin\alpha$.

20. If $\frac{a}{r} = \theta^2 + b$, find $\frac{d^2u}{d\theta^2}$.

II. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. Find the partial differential equation of all spheres of radius 3 units having their centre on the xy-plane.

2. Solve, $g(p^2 z + q^2) = 4$.

3. Solve, $z(p^2 - q^2) = x + y$.

4. Solve, $q - xp - p^2 = 0$ by Charpit's method.

5. Solve, $[D^2 + 2DD' + (D')^2]Z = e^{2x+3y}$.

III. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. Using the method of separation of symbols prove that $u_1x + u_2x^2 + u_3x^3 + \cdots$ to ∞ $= \frac{x}{1-x}u_1 + \frac{x^2}{(1-x)^2}\Delta u_1 + \frac{x^3}{(1-x)^3}\Delta^2 u_1 + \cdots$.

2. Find the third divided difference of the function $f(x) = x^3 - 2x$, for the arguments 2, 4, 9, 10, ...

3. Find a polynomial in 'x' from the table.

x	0	1	2	3
y	2	3	12	35

4. Obtain an approximate value of $\log_{10} 2$ by calculating $\int_0^1 \frac{dx}{1+x}$ using Simpson's 3/8th rule after dividing the range into eight equal parts.

5. Find $f'(1)$ and $f''(1)$ from the table.

x	0	1	2	3	4	5
y	4	8	15	7	6	2

IV. Answer any **FOUR** of the following : **(4 × 5 = 20)**

1. Show that $\vec{F} = r^5\vec{r}$ is conservative and find the potential function.

2. A particle moving with SHM has speeds 8 and 6 at distances 3 and 4 respectively from the mean positions. Find the period and the maximum acceleration.

3. Derive the expression for path of a projectile and show that it represents a parabola.

4. A particle is projected with velocity of 80 ft/sec at an angle of 45° to the horizontal. Find its range on a plane inclined at an angle of 30° to the horizontal when projected up the plane.

5. If the tangential and normal acceleration of a particle describing a plane curve by constant throughout, show that the radius of curvature is given by $\rho = (at+b)^2$.

6. A particle is projected horizontally with speed $\frac{\sqrt{ag}}{\sqrt{2}}$ from the highest point outside a fixed smooth sphere of radius 'a. Show that it will leave the sphere at the point whose vertical distance below the point of projection is $\frac{a}{6}$.

V. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. If the path of a particle is $r = a\tan\theta$ and the acceleration is directed towards the origin, show that the acceleration is $\frac{k^2}{r^3}\left[3+\frac{2a^2}{r^2}\right]$.

2. Derive the differential equation of a central orbit in the form $f = \frac{h^2}{p^3}\frac{dp}{dr}$.

3. A particle describes the curve $au = \tanh\left(\frac{\theta}{\sqrt{2}}\right)$ find the law of force.

4. A particle describe a circle $r = 2a\sin\theta$ with a speed $k\,\text{cosec}^2\theta$. Show that its acceleration is wholly along the joining the particle to the origin.

Q.P. Code – 42533

Fifth Semester B.Sc. Degree Examination, October/November 2019

(CBCS Scheme)

Mathematics

Paper 5.1 – ADVANCED ALGEBRA AND NUMERICAL METHODS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1. *Answers ALL the questions.*
2. *Answer should be written completely in English.*

PART – A

I. Answer any **SIX** of the following : **(6 × 2 = 12)**

1. Define skew field. Give an example for a skew field.

2. Show that $S = \left\{ \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} \middle/ a, b \in z \right\}$ is a subring of $M_2(Z)$.

3. If I be a ideal of a ring R with unity $1 \in I$ then prove that $I = R$.

4. Define vector space over a field F.

5. Prove that the set of vectors of $V(F)$ containing the zero vector is linearly dependent.

6. Solve the equation $x^3 - 4x - 9 = 0$ in (2, 3) by bisection method in two steps.

7. Explain Jacobi-iteration method to solve the system of three equations.

PART – B

II. Answer any **SIX** of the following : **(6 × 3 = 18)**

8. Let $(R, +, \cdot)$ be a ring $\forall\, a \in R$, then prove that $a \cdot 0 = 0 \cdot a = 0$ where 0 is the additive identity.

9. Show that $S = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \middle/ a, b \in Z \right\}$ is neither left ideal nor right ideal.

10. Prove that Kernel of homomorphism of ring is a subring.

11. Show that $W = \{(x,0,0)/x \in R\}$ is a subspace of $V_3(R)$.

12. If $T: V_3(R) \to V_3(R)$ defined by $T(x,y,z) = (0,y,z)$ then show that T is a linear transformation.

13. Find the cube root of 54 correct to three places of decimal by Newton-Raphson method.

14. Solve the equations $10x + y + z = 12$, $2x + 10y + z = 13$, $2x + 2y + 10z = 14$ by Gauss-Seidal iteration method up to three iterations.

PART – C

III. Answer any **FOUR** of the following : **(4 × 5 = 20)**

15. Prove that the ring of integers module 'n' $(z_n, +_n, X_n)$ is an integral domain if and only if 'n' is a prime number.

16. Let $(R, +, \cdot)$ be a ring, a non empty set S of a ring R is a subring of R then prove that

(a) $\forall\, a, b \in S \Rightarrow a - b \in s$

(b) $\forall\, a, b \in S \Rightarrow a \cdot b \in s$.

17. If $f: R \to R'$ be a homomorphism with Kernel k then prove that f is one-one iff $K = \{0\}$.

18. If I be an ideal of a ring R then prove that

(a) R is a commutative then R/I is also commutative

(b) If R has unity then R/I is also unity.

19. State and prove fundamental theorem of homomorphism of rings.

PART – D

IV. Answer any **FOUR** of the following : **(4 × 5 = 20)**

20. Prove that the intersection of any two subspaces of a vector space is also a subspace but the union of two subspaces need not be subspace.

21. Express the vectors $(2,-5,-1)$ as a linear combination of the vectors $(1,2,3)$, $(2,1,1)$ and $(1, 3, 2)$ of $V_3(R)$.

22. Prove that in an n-dimensional vector space $V(F)$

(a) any $(n+1)$ vectors of V are linearly dependent

(b) no set of $(n-1)$ elements can span V.

23. Find the linear transformation $T: R^2 \to R^3$ such that $T(-1,1) = (-1,0,2)$ and $T(2,1) = (1,2,1)$.

24. Find the range space, null space, rank and nullity of a linear transformation $T: V_3(R) \to V_2(R)$ defined by $T(x,y,z) = (y-x,\ y-z)$.

PART – E

V. Answer any **FOUR** of the following : **(4 × 5 = 20)**

25. Solve the equation $x^3 - x^2 - x - 3 = 0$ over (2, 2.5) by Newton-Raphson method correct to three places of decimal.

26. Solve the equation $x^4 - x - 10 = 0$ has one root between 1.8 and 2 correct to three places of decimal by Regula-Falsi method.

27. Solve the following equations by Gauss elimination method.

$2x_1 + 4x_2 + x_3 = 3$, $3x_1 + 2x_2 - 2x_3 = -2$ and $x_1 - x_2 + x_3 = 6$.

28. Apply Euler's modified method to find y for x = 0.05 for the equation $\frac{dy}{dx} = x + y$ with $y(0) = 1$.

29. Apply Runge-Kutta method to solve the equation $\frac{dy}{dx} = 1 + \frac{y}{x}$ with $y(2) = 2$ for $x = 2.1$.

Q.P. Code – 42534

Fifth Semester B.Sc. Degree Examination, October/November 2019

(CBCS Semester Scheme)

Mathematics

Paper 5.2 (A) – ANALYSIS AND INTEGRAL TRANSFORMS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1. *Answers ALL questions.*
2. *Answer should be written completely in English.*

PART – A

I. Answer any **SIX** questions : **(6 × 2 = 12)**

1. Evaluate : $\int_0^\infty x^2 e^{-x^3} dx$ using Gamma function.

2. Evaluate $\beta(5.5, 3)$.

3. Define half-range cosine series in $(0, L)$.

4. Evaluate : $L\{\sin^2 t\}$.

5. Evaluate : $L^{-1}\left\{\frac{1}{S(S^2+3)}\right\}$.

6. Define inverse Fourier sine transform of $F_S(\alpha)$.

7. Prove that $F[f(ax)] = \frac{1}{a}F\left(\frac{u}{a}\right)$.

PART – B

II. Answer any **SIX** questions : **(6 × 3 = 18)**

8. Show that $\beta(n,m) = \beta(m,n)$.

9. Evaluate $\int_0^\infty \frac{x^6(1-x^8)}{(1+x)^{22}} dx$.

10. State Dirichlet's condition.

11. Find $L\{F(t)\}$ if $F(t)=\begin{cases}0, & 0\le t\le 4\\ 2t, & t\ge 4\end{cases}$.

12. Find $L\{t\cos t\}$.

13. Prove that $f(x)=\frac{1}{\pi}\int_0^\infty\int_{-\infty}^\infty f(t)\cos(\mu(t-x))dt\,du$.

14. Prove that $F_S[f'(x)]=-\alpha F_C[f(x)]$.

PART – C

III. Answer any **FOUR** questions : **(4 × 5 = 20)**

15. With the usual notation prove that $\sqrt{(1/2)}=\sqrt{\pi}$.

16. Using Beta and Gamma function, evaluate $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}dx \times \int_0^1 \frac{dx}{\sqrt{1+x^4}}$.

17. Obtain Fourier series for $f(x)=\begin{cases}1+2x & -3\le x\le 0\\ 1-2x & 0\le x\le 3\end{cases}$ in (-3, 3).

18. Obtain the Fourier series for $f(x)=e^{-ax}$ in $(0, 2\pi)$.

19. Obtain the Fourier half range sine series for $f(x)=x(\pi-x)$ over $(0,\pi)$. Hence deduce that $\frac{\pi^2}{6}=\sum_{n=1}^{\infty}\frac{1}{n^2}$.

IV. Answer any **FOUR** questions : **(4 × 5 = 20)**

20. If $L\{f(t)\}=F(s)$, then prove that

(a) $L\{e^{at}f(t)\}=F(s-a)$

(b) $L\{f(at)\}=\frac{1}{a}F\left(\frac{s}{a}\right)$.

21. Evaluate :

(a) $L\{\sin 3t \cos 2t\}$

(b) $L\{(t+2)e^{-5t}\}$.

22. Evaluate $L^{-1}\left\{\dfrac{s+8}{s^2+4s+6}\right\}$.

23. Evaluate $L^{-1}\left\{\dfrac{1}{s(s^2+9)}\right\}$ using convolution theorem.

24. Solve $y''+4y=3$, given $y(0)=0=y'(0)$ using Laplace transforms.

V. Answer any **FOUR** questions : **(4 × 5 = 20)**

25. Find the Fourier integral expansion of $f(x)=\begin{cases}1 \text{ for } & 0\le x\le \pi \\ 0 \text{ for } & x>\pi\end{cases}$. Hence evaluate $\int_0^\infty \left(\dfrac{1-\cos \pi u}{u}\right)\sin ux\, dx$.

26. Obtain the Fourier transform of $f(x)=e^{-a^2x^2}$, where a is a constant.

27. Find the function $f(x)$ whose cosine transform is given by $F_C[f(x)]=\begin{cases} a-\dfrac{\alpha}{2}, & 0\le \alpha\le 2a \\ 0\ , & \alpha\ge 2a\end{cases}$.

28. Show that $xe^{\frac{-x^2}{2}}$ is self reciprocal w.r.to Fourier sine transformation.

29. Find the finite Fourier cosine transform of $f(x)=\sin nx$.

Q.P. Code – 42534

Fifth Semester B.Sc. Degree Examination,
October/November 2018

(CBCS – Semester Scheme)

Mathematics

Paper 5.2(a) — ANALYSIS AND INTEGRAL TRANSFORMS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1. *Answer ALL questions.*
2. *Answers should be written completely in English.*

PART – A

I. Answer any **SIX** questions : **(6 × 2 = 12)**

1. Define Gamma and Beta functions.

2. Evaluate : $\frac{\Gamma(7)}{2\Gamma(4)\,\Gamma(3)}$.

3. Define Fourier series.

4. Find $L\{\sin 4t \cdot \cos 2t\}$.

5. If $L\{f(t)\} = F(S)$ then prove that $L\{f'(t)\} = SF(S) - f(0)$

6. Write down the Fourier cosine and sine transforms of $f(x)$.

7. If a and b are any two constants and $f(x)$ and $g(x)$ are any two function then prove that

   $F[a\,f(x) + b\,g(x)] = a\,F[f(x)] + b\,F[g(x)]$.

PART – B

II. Answer any **SIX** questions : **(6 × 3 = 18)**

1. Evaluate : $\Gamma(-5/2)$.

2. Show that $\int_0^\infty e^{-x^3}\,dx = \frac{1}{3}\Gamma(1/3)$.

3. Define half-range sine and cosine series of $f(x)$ in $[0, L]$.

4. Find : $L\{(t+3)^2 e^{5t}\}$

5. Find : $L^{-1}\left\{\frac{S+2}{S^2-4S+13}\right\}$

6. Find the Fourier sine transform of $f(x) = e^{-ax}$, $a > 0$

7. Find the inverse complex Fourier transform of $f(x) = e^{-a|\alpha|}$, $a > 0$.

PART – C

III. Answer any **FOUR** questions : **(4 × 5 = 20)**

1. Show that $\beta(m, n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$ $m, n > 0$

2. Evaluate : $\int_0^{\pi/2} \cos^5 \theta \cdot \sin^2 \theta \, d\theta$

3. Find the Fourier series for the function $f(x) = |x|$ in the interval $(-\pi, \pi)$ and hence deduce that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cdots$

4. Obtain the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$. Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

5. Obtain the Fourier half range sine series for $f(x) = x$ in $(0, \pi)$.

IV. Answer any **FOUR** questions : **(4 × 5 = 20)**

6. Find :

   (a) $L\{\cos^3 t\}$

   (b) $L\{(t+1)^3\}$

7. Find :

(a) $L\{t\,e^{-t}\cosh t\}$

(b) $L\left\{\dfrac{2\sin 5t\cdot\cos 3t}{t}\right\}$

8. Find : $L^{-1}\left\{\dfrac{S^2}{(S-1)(S^2+1)}\right\}$

9. Using convolution theorem find $L^{-1}\left\{\dfrac{1}{(S+2)(S+4)}\right\}$

10. Using Laplace Transform solve $\dfrac{d^2y}{dt^2}-3\dfrac{dy}{dt}+2y=e^{3t}$ given that $y(0)=0$ and $y'(0)=0$.

V. Answer any **FOUR** questions : **(4 × 5 = 20)**

11. Find the Fourier integral expansion of

$$f(x)=\begin{cases}0 & x<2\\ 1 & 2<x<3\\ 0 & x>3\end{cases}$$

12. Find the complex Fourier transform of

$$f(x)=\begin{cases}1-|x| & \text{when } |x|\le 1\\ 0 & \text{when } |x|>1\end{cases}$$

13. If a is any non-zero constant and $F[f(x)]=\hat{f}(\alpha)$ then prove that

$$F[f(ax)]=\frac{1}{|a|}\hat{f}\left(\frac{\alpha}{a}\right).$$

14. Find the finite Fourier sine and cosine transform of $f(x)=x^2 \quad 0<x<4$.

15. Prove that

(a) $F_c[f'(x)]=\dfrac{-\sqrt{2}}{\sqrt{\pi}}f(0)+\alpha\,F_s[f(x)]$

(b) $F_c[f''(x)]=\dfrac{-\sqrt{2}}{\sqrt{\pi}}f'(0)-\alpha^2\,F_c[f(x)]$

Q.P. Code – 22533

Fifth Semester B.Sc. Degree Examination,
October/November 2019

(Semester Scheme)

Paper V – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : 1) Answer All questions.

2) Answer should be written completely in English.

I. Answer any **FIFTEEN** of the following : **(15 × 2 = 30)**

1. Define an integral domain and give an example.

2. Find all zero divisors of the ring (z_8, t_8, x_8).

3. Define homomorphism of rings.

4. Let R be a ring and 'a' be a fixed element of R. Show that $S = \{r \in R / ar = 0\}$ is a subring of R.

5. In a ring $(R, +, \cdot)$, prove that $(-1)a = a$, $\forall a \in R$ and '1' is the unity of 'R'.

6. If 'I' is an ideal of a ring 'R' with unity and $1 \in I$, then prove that $I = R$.

7. If $\vec{r} = t\hat{i} - t^2\hat{j} + \sin t\,\hat{k}$, then find $\dfrac{d^2\vec{r}}{dt^2}$ at $t = 0$.

8. Find the unit tangent vector for the space curve $\vec{r} = 3\cos t\,\hat{i} + 3\sin t\,\hat{j} + 4t\,\hat{k}$.

9. Define the radius of curvature and torsion of curve.

10. Find the equation of tangent plane to the surface $z = x^2 + y^2$ at (1, 1, 2).

11. Find the unit normal vector to the surface $x^2 + 2y - 3z = 5$ at (1, 2, 0) on it.

12. Find the Cartesian co-ordinates of the point whose spherical-polar coordinates are $\left(2, \dfrac{3\pi}{4}, \dfrac{\pi}{6}\right)$.

13. Show that $grad(\vec{r} \cdot \vec{a}) = \vec{a}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

14. If $\vec{F} = 3y^4z^2\hat{i} + 4x^2z^2\hat{j} - 3x^2y^2\hat{k}$ is a solenoidal

15. If $\vec{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$, then find curl $\vec{F}$.

16. Find $\nabla^2\phi$, if $\phi = x^2 - y^2 + z$.

17. Write the Legendre's differential equation.

18. Prove that $p_n(-x) = (-1)^n p_n(x)$

19. Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cdot \cos x$.

20. Show that $\frac{d}{dx}(xJ_1(x)) = x \cdot J_0(x)$.

II. Answer any **FOUR** of the following. **(4 × 5 = 20)**

21. Prove that every finite integral domain is field.

22. Show that the set of all matrices of the form $\begin{pmatrix} 0 & x \\ 0 & y \end{pmatrix}$. $x, y \in Q$ is a non-commutative ring without unity, the binary operation being addition and multiplication of matrices.

23. Prove that the intersection of any two subrings is also a subring.

24. If $f: R \to R'$ is an isomorphism and 'R' is a field, then show that R' is also a field.

25. Let 'a' be an element of a commutative ring 'R', then prove that $aR = \{ar / r \in R\}$ is an ideal of 'R'.

26. State and prove the fundamental theorem on homomorphism of rings.

III. Answer any **THREE** of the following. **(3 × 5 = 15)**

27. Derive Serret-Frenet formula for the curve $\vec{r} = \vec{r}(s)$.

28. For the space curve $x = a\cos u$, $y = a\sin u$, $z = bu$, show that the curvature is $\frac{a}{a^2+b^2}$ and torsion is $\frac{b}{a^2+b^2}$.

29. Show that the Serret-Fernet formulae can be written in the form $\vec{t}^{\,1} = \vec{d} \times \vec{t}$, $\vec{n}^{\,1} = \vec{d} \times \vec{n}$, $\vec{b}^{\,1} = \vec{d} \times \vec{b}$ and determine the vector $\vec{d}$.

30. Find the equation of the tangent plane and normal line to the surface $x^2 + y^2 = z + 3$ at (2, -1, 2).

31. Prove that cylindrical polar coordinate system is an orthogonal curvilinear coordinate system.

IV. Answer any **THREE** of the following. **(3 × 5 = 15)**

32. Find the angle between the surfaces $3x^2 - 2y^2 - 3z^2 + 8 = 0$ and $4x^2 + 9y^2 = 40z$ at (-1, 2, 1).

33. Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$, where $r^2 = x^2 + y^2 + z^2$.

34. Show that $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational. Find ϕ such that $\vec{F} = \nabla\phi$.

35. Prove that $\text{div}(\phi \vec{f}) = \phi \,\text{div}\, \vec{f} + \vec{f} \cdot \text{grad}\phi$, where ϕ is scalar function and $\vec{f}$ is vector function.

36. Show that the vector $\frac{\vec{r}}{r^3}$ is both solenoidal and irrotational, where $\vec{r}$ is the position vector of $p(x, y, z)$.

V. Answer any **TWO** of the following : **(2 × 5 = 10)**

37. Show that $\frac{1+t}{\sqrt{1-2xt+t^2}} = 1+\sum_{n=0}^{\infty}\left[p_n(x)+p_{n+1}(x)\right]t^{n+1}$.

38. State and prove Rodrigue's formula.

39. Prove that $\frac{d}{dx}\left(x^{-n}J_n(x)\right) = -x^{-n}J_{n+1}(x)$.

40. Show that $np_n(x) = xp_n'(x) - p_{n-1}'(x)$.